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# Geometric Models, Antenna Gains, and Protection Ratios as Developed for BC SAT-R2 Conference Software



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GEOMETRIC MODELS, ANTENNA GAINS, AND PROTECTION RATIOS  
AS DEVELOPED FOR BC SAT-R2 CONFERENCE SOFTWARE

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SUMMARY

This paper gives an overview of mathematical models used in the software package developed for use at the 1983 Regional Administrative Radio Conference on broadcasting satellites. The models described are those used in the Spectrum Orbit Utilization Program (SOUP) analysis. The geometric relationships necessary to model broadcasting satellite systems are discussed. Antenna models represent co-polarized and cross-polarized performance as functions of the off-axis angle. The protection ratio is modelled as a co-channel value and a template representing systems with frequency offsets.

INTRODUCTION

The computer software that has been developed for use at the 1983 Regional Administrative Radio Conference incorporates a series of mathematical models to represent broadcasting satellite systems. The various models are combined by the software to allow the synthesis and analysis of proposed systems. Based upon a set of specified system characteristics (including desired signal-to-noise ratio, antenna characteristics, geographical coverage areas, and receiver characteristics), the software can determine required satellite powers, satellite positions, channel assignments, and carrier-to-interference ratios. Repeated use of these software models can lead to the specification of systems of broadcasting satellites that may meet the requirements of the administrations of Region 2. This paper describes some of the mathematical models used to represent the elements of broadcasting satellite systems. A brief description of the modelling techniques used and the inherent assumptions are given. The goal of the paper is to impart a general understanding of the models and to indicate the flexibility of the models to represent whatever characteristics might be decided upon by the Conference. Consequently, complex equations are not discussed in detail. Full documentation of the models is available in reference 1. Unless otherwise stated, the models described are those used in the analysis program called the Spectrum Orbit Utilization Program (SOUP).

The models implemented in the software are fully consistent with the Conference Preparatory Meeting (CPM) report (ref. 2), and additionally have the flexibility to represent other characteristics such as those in the Consultative Committee on International Radio (CCIR) reports and those used at WARC - 77.

## GEOMETRIC MODELS

There are a number of calculated geometric quantities that are necessary to model broadcasting satellites operating in geostationary orbits. These include ranges, angles, and latitudes and longitudes of antenna aim points, for both downpaths and feederlinks. Fundamental assumptions and constants used are given in table I. A sketch of the geometry for the transmit and receive earth stations in the equatorial plane is shown in figure 1.

### Feederlink Calculations

From each feederlink earth station (FES) location, the following quantities are calculated:

- (1) Slant range  $R$ , and elevation angle  $\epsilon$  to the desired satellite.
- (2) Slant range  $R$ , elevation angle, and off-axis angle  $\varphi_1$  to every other satellite where interference may occur (out to the second adjacent channel. (see fig. 2)).

From each satellite location, for each beam, the following quantities are calculated:

- (1) Latitude and longitude of the antenna boresight (calculated in BEAM-FIT program).
- (2) Off-axis angle  $\varphi_2$  to every transmit ground point in the desired feederlink service area and to every other transmit ground point from which interference may come.
- (3) Orientation angle of antenna beam ellipse major axis relative to a line determined partly by a projection of the ground point, for each ground point, of concern. This angle allows calculation of the gain of an elliptical beam satellite antenna in the direction of a specific ground point.

### Downpath Calculations

From each satellite location, for each beam, the following quantities are calculated:

- (1) Latitude and longitude of the antenna boresight (calculated in BEAM-FIT program).
- (2) Slant range  $R$  and off-axis angle  $\varphi_2$  toward every ground receive point in the desired service area and toward every other ground receive point which may be subject to interference (similar to fig. 2).
- (3) Orientation angle of antenna beam major axis, similar to the calculation used on the feederlink.

For each earth station receive (ESR) test point the following quantities are calculated:

- (1) Elevation angle  $\epsilon$  to the desired satellite.
- (2) Off-axis angle  $\varphi_1$  to every other satellite which may cause interference (similar to fig. 2).

### General Remarks on Geometric Calculations

If a ground point and a satellite are determined to be over the horizon with respect to each other, no subsequent interference calculations are performed.

Antenna pointing errors are taken into account by increasing the off-axis angle by pointing error when carrier signals are calculated, and by decreasing the off-axis angle when interferences are calculated. This results in conservative or worst-case calculations of carrier-to-interference (C/I) ratios.

Rotation errors of elliptical antennas are not modelled.

A more complete description of the geometric models is given in Chapter III of reference 1.

## ANTENNA GAINS

For the calculation of carrier-to-interference (C/I) ratios in broadcasting satellite systems, several characteristics of the transmit and receive antennas are required. These include the beamwidth, the on-axis gain, and the off-axis gain as a function of the off-axis angle. The four different antennas in the system (i.e., ground transmit, spacecraft receive, spacecraft transmit, and earth station receive antennas) can each be modelled in a form flexible enough to accommodate any decision that might be reached by the Conference. The following sections describe the main feature of those models.

### Antenna Beamwidth

The half-power beamwidth of an antenna is defined as the angular measure of the width of the antenna beam at the half power (-3 dB) points. This quantity ( $\varphi_0$ ) is expressed in radians or degrees.

The half-power beamwidth is related to the antenna diameter by the following expressions:

$$\varphi_0 = \frac{223}{180} \frac{\lambda}{D}, \text{ rad}$$

$$\text{or } \varphi_0 = \frac{21.28}{fD}, \text{ deg}$$

where

$\lambda$  wavelength, m  
 $D$  antenna diameter, m  
 $f$  frequency, GHz

The satellite antenna beamwidths calculated by the program BEAMFIT are the beamwidths at a specified gain contour of  $\Delta G$  dB. For an elliptical antenna, both the major and minor axis beamwidths are calculated. Internal to the analysis program SOUP 5, all satellite antenna beamwidths are converted to 3 dB beamwidths by the following formula:

$$\varphi_0 = \varphi'_0 \sqrt{3/\Delta G}$$

where

$\varphi_0$  3 dB (half-power) beamwidth, rad or deg  
 $\varphi'_0$  beamwidth at contour  $\Delta G$ , rad or deg  
 $\Delta G$  gain contour specified in BEAMFIT program, an input variable, dB

For the analysis program, all ground station antenna beamwidths must be specified as 3 dB (half-power) beamwidths. For ground antennas, the software accepts either beamwidth or diameter as the antenna specification. Spacecraft antennas must be described by their beamwidths.

Spacecraft antennas may be limited in size due to packaging and cost constraints. The analysis model has minimum antenna beamwidth limits for both spacecraft transmit and receive antennas. Presently, these limits are set to  $0.8^\circ$ .

#### On-Axis Gains

The on-axis gain  $G_0$  of an antenna is specified as the maximum power gain of the antenna relative to an isotropic radiator. The direction of maximum gain is known as the boresight axis.

The on-axis gain is given by

$$G_0 = \text{EAP} \left( \frac{\pi}{\varphi_0} \frac{223}{180} \right)^2 \cdot \text{AXR}$$

$$\text{GAO} = 10 \log_{10} G_0$$

where

$G_0$  numerical on-axis gain  
 $\text{EAP}$  antenna aperture efficiency, ( $0 < \text{EAP} < 1$ )  
 $\varphi_0$  antenna beamwidth, rad  
 $\text{AXR}$  axial ratio (major/minor) for an elliptical antenna.  $\varphi_0$  is the major axis beamwidth for an elliptical antenna.  
 $\text{GOA}$  on-axis gain, dB

## Cross-Polarized Gains

The previous sections have discussed the performance of antennas operating with copolarized signals. When an antenna operates with cross-polarized signals, it generally has lower gain to these signals and hence provides a discrimination against cross-polarized signals. Details of antenna performance with cross-polarized signals are given in later sections.

## Off-Axis Gain

In general, points in a communication system do not lie exactly on the boresight axes of the antennas, where the antenna gains are at their maximum values. Performance measurements of many antennas have resulted in the development of mathematical models to represent antenna performance at angles off the boresight. Figure 3 is a graph of the model suggested by the CPM for the feederlink transmit antenna. The following sections describe models available for the several antennas required in broadcasting satellite systems.

### Earth Station Transmit (EST) Antenna

The co- and cross-polarized gain characteristics suggested by the CPM for this antenna are shown in figure 3. The equations describing this antenna are:

#### Co-polar component (dBi relative to isotropic source)

$$36 - 20 \log \varphi \quad \text{for } 0.1^\circ \leq \varphi < 46.6 \lambda/D_{\min}$$

$$8 + 20 \log (D_{\min}/\lambda) - 2.5 \times 10^{-3} (D_{\min} \varphi/\lambda)^2$$

$$\text{for } 46.6 \lambda/D_{\min} \leq \varphi < \text{junction with next segment}$$

$$29 - 25 \log \varphi \quad \text{for junction with previous segment} \leq \varphi < 36^\circ$$

$$-10 \quad \text{for } \varphi \geq 36^\circ$$

#### Cross-polar component (dBi relative to isotropic source)

$$G_{\max} - 30 \quad \text{for } \varphi < 35 \lambda/D$$

$$9 - 20 \log \varphi \quad \text{for } 35 \lambda/D \leq \varphi < 8.7^\circ$$

$$-10 \quad \text{for } \varphi \geq 8.7^\circ$$

where

$D_{\min}$  diameter of the smallest antenna to be accommodated in the plan, 2.5 m

$D$  diameter of the antenna used,  $\geq 2.5$  m

$G_{\max}$  on-axis co-polar gain of the antenna used =  $8 + 20 \log (D/\lambda)$   
(efficiency  $\approx 65$  percent)

In the software model of this antenna,  $\theta_{min}$  is an input variable, and thus can easily be changed to any value decided upon by the Conference. The EST antenna is assumed to have a circular beam.

### Satellite Receiving Antenna

The CPM recommended, for planning, that the broadcast-satellite receiving antenna reference patterns be identical to the broadcast-satellite transmitting antenna patterns except for degraded cross-polarization characteristics inside the main beam.

The software implemented can accept independent specifications of satellite receive and transmit antenna characteristics. Thus the model can operate with any choice of satellite receiving antenna characteristics. This antenna may have an elliptical beam.

### Satellite Transmitting Antenna

The CPM discussed two models for this antenna. They are shown in figures 4 and 5.

The following equations were recommended by the CPM for the general reference pattern corresponding to figure 5.

$$\begin{aligned}
 G(\varphi/\varphi_0) &= G_0 - f \\
 f &= 12 (\varphi/\varphi_0)^2 && \text{for } 0 \leq \varphi/\varphi_0 \leq 0.5 \\
 f &= 18.75 \varphi_0^2 [(\varphi/\varphi_0) - x]^2 && \text{for } (0.40/\varphi_0) + x < \varphi/\varphi_0 \leq (1.155/\varphi_0) + x \\
 f &= 25 && \text{for } (1.155/\varphi_0) + x < \varphi/\varphi_0 \leq (1.60/\varphi_0) + x \\
 f &= 17.5 + 25 \log ((\varphi_0/0.8)[(\varphi/\varphi_0) - x]) && \text{for } (1.60/\varphi_0) + x < \varphi/\varphi_0 \leq (4.0/\varphi_0) + x \\
 f &= 35 && \text{for } (4.0/\varphi_0) + x < \varphi/\varphi_0 \leq (6.97/\varphi_0) + x \\
 f &= 11.5 + 25 \log ((\varphi_0/0.8)[(\varphi/\varphi_0) - x]) && \text{for } (6.97/\varphi_0) + x < \varphi/\varphi_0 \\
 f &= G_0 && \text{for } f > G_0
 \end{aligned}$$

where

$\varphi$	off-axis angle, deg
$\varphi_0$	half-power beamwidth of main lobe, deg, ( $\varphi_0 \geq 0.80$ )
$G(\varphi/\varphi_0)$	gain as function of off-axis angle, dB
$G_0$	on-axis gain, dB
$f$	relative gain, dB below on-axis gain
$x$	$0.5 [1 - 0.8/\varphi_0]$ by definition

Figure 5 shows a plot of the recommended reference pattern for the case when  $\varphi_0 = 2^\circ$  and  $G_0 = 38.8$  dB.

The models of both figures 4 and 5 have been implemented.



## Earth Station Receiving (ESR) Antenna

The CPM discussed ESR antennas of the types shown in figures 6 and 7. Both of these models have been implemented. The ESR antenna is assumed to have a circular beam.

### Software Implementation of the Models

The off-axis gain for a particular antenna may be represented by the following function:

$$G(\varphi) = GOA - f$$

where

$G(\varphi)$     off-axis gain, dB, as a function of the off-axis angle,  $\varphi$   
GOA        on-axis gain, dB  
 $f$          gain fall-off function, dB

The function  $f$  is represented by different mathematical functions over different domains of the variable  $\varphi$ . Thus, the segments of an arbitrary off-axis gain function (fig. 7, for example) can easily be represented mathematically. In the software,  $f$  is typically a constant, a polynomial, an exponential function, or a logarithmic function. In the analysis program SOUP 5, the off-axis gain is computed as a numerical quantity. A more complete description of the antenna gain modelling is given in Chapter IV of reference 1.

In the case of an elliptical spacecraft antenna, the off-axis gain is calculated by first determining an equivalent 3 dB beamwidth in the off-axis direction. Then the gain is computed by using that 3 dB beamwidth in the appropriate antenna gain model.

### PROTECTION RATIOS

A key technical parameter that needs to be specified in planning broadcasting-satellite systems is the television protection ratio defined as the ratio at the receiver input of the wanted signal to the aggregate interfering signal power required to meet a subjectively assessed picture quality.

The required orbital spacing between adjacent broadcasting satellites and the optimal frequency spacing between adjacent television channels in a plan will be dependent primarily on the co-channel and adjacent channel (i.e., frequency off-set) protection ratio requirements that are adopted for planning purposes.

The analysis program SOUP 5 can accommodate the protection ratio models discussed in the CPM report. The following discussion applies to interference between two frequency modulated television signals.

## Co-channel Protection Ratio

The co-channel protection ratio  $PR_0$  is the carrier-to-interference ratio required when all the interferers are co-channel (i.e., operating at the same frequency). Previous CCIR work and inputs to the CPM have indicated that  $PR_0$  is a function of the frequency deviation of the wanted and interfering signals, the desired quality of the wanted signal, the types of television pictures being transmitted, and possibly the signal-to-noise ratio of the wanted signal. Consideration of all these factors leads to a determination or choice of a specific value for  $PR_0$ . The developed software has  $PR_0$  as an input parameter. Separate protection ratios can be specified for the feederlink, the downpath, and the entire system, which combines both the feederlink and downpath interferences. Numerical values used for the downpath have been in the range 25 to 30 dB.

To account for multiple interferers, the interfering signals are assumed to add on a power basis.

## Protection Ratios at Offset Frequencies

For cases other than co-channel interference, the protection ratio is defined by a template which gives the fall-off (in dB) from the co-channel value as a function of normalized frequency offset. The normalized frequency offset is the frequency offset divided by the Carson's rule bandwidth (peak-to-peak deviation + 2 x top-baseband-frequency). The protection ratio template proposed by the CPM is shown in figure 8. The frequency offset is one channel separation for upper and lower adjacent channel interference, and two channel separations for second upper and second lower adjacent channel interference.

In the software package developed for use at the Conference, the template is modelled as a series of straight line segments. The number of segments can be as great as ten. The frequency offsets at the breakpoints of the template are input variables. The template is not required to be symmetric.

## General Remarks on Protection Ratios

The co-channel and offset frequency protection ratios discussed in this paper were determined for frequency modulated television signals. It is possible that the Conference might decide to plan for other types of signals, such as for sound broadcasting or high definition television, which might require different protection ratios. The analysis software could properly treat such systems merely by the input specification of the appropriate  $PR_0$  and template.

Several administrations are currently conducting measurements that may suggest the use of a different protection ratio template for frequency modulated television. Any revised template could easily be accommodated merely by changing the values of the input variables that describe the template segments and their associated domains.

In a complete interference environment, there may exist simultaneously multiple co-channel interferers as well as multiple interferers at the adjacent

and second adjacent channels. The combination of these multiple interferers is treated in another paper under the subjects "Aggregate and Total Interference" (ref. 3).

The model used for the protection ratio template has no limits on the domain of the normalized frequency offset (as suggested by the CPM). However, the protection ratio at the second adjacent channel is typically below 0 dB and seems to have little practical impact on planning.

#### REFERENCES

1. Davidson, J. H., Ottey, H. R., Sawitz, P., and Zusman, F. S.: "SOUP 5 Version 3, Technical Manual," ORI Technical Report 2053, March 15, 1983.
2. Technical Bases for the Regional Administrative Radio Conference 1983 for the Planning of the Broadcasting-Satellite Service in Region 2, Report of the CCIR Conference Preparatory Meeting, Joint Meeting, Study Groups 4,5,9,10 and 11, Geneva, 28 June - 9 July 1982. International Telecommunications Union, 1982.
3. Sawitz, P.: Rain Attenuation and Depolarization Channelizations - Aggregate and Total Interference Calculations. Paper SEM-LIMA-5.3, BC-SAT-R2 Conference, Lima, Peru, April 18-22, 1983.

TABLE I. - FUNDAMENTAL ASSUMPTIONS AND CONSTANTS FOR  
GEOMETRIC CALCULATIONS

Symbol	Item	Value
	Spherical Earth:	
$R_E$	Earth radius, m	$6.3800 \times 10^6$
$R_O$	Radius of geostationary orbit, m	$6.6094 \times R_E$
		$4.2168 \times 10^7$
$H$	Height of satellite above equator, m	$3.5788 \times 10^7$

<sup>a</sup>Alternate value.

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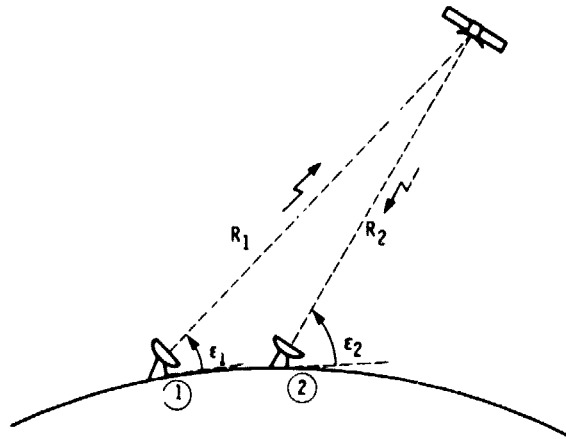


Figure 1. - Simplified geometry of satellite relative to feederlink earth station 1 and receive earth station 2.

$R$  = slant range  
 $\epsilon$  = elevation angle

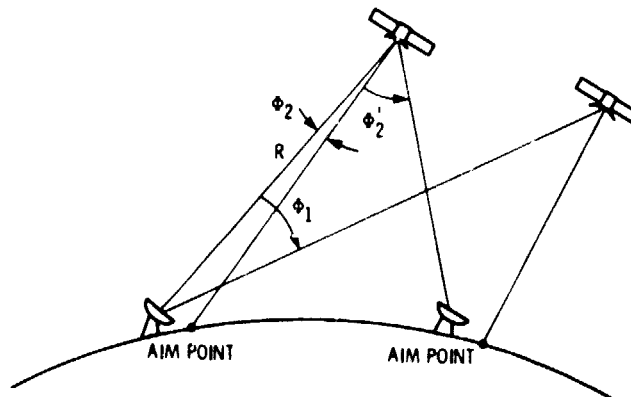


Figure 2. - Simplified geometry for calculation of range ( $R$ ) and off-axis angles ( $\phi$ ).

$\phi_1$  - Off-axis angle to non-target satellite.  
 $\phi_2$  - Off-axis angle to test point in desired service area.  
 $\phi_2'$  - Off-axis angle to test point in non-target service area.

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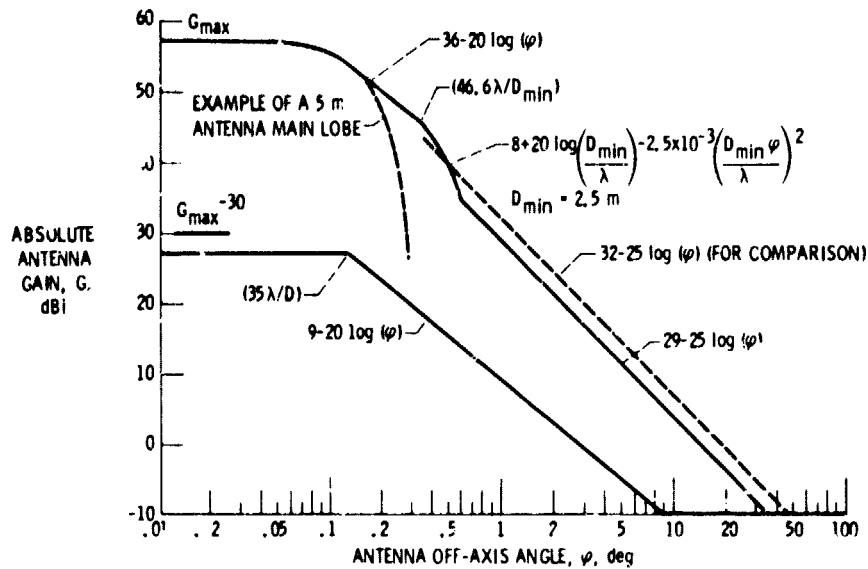


Figure 3. - Suggested antenna reference patterns for planning of the feeder links.

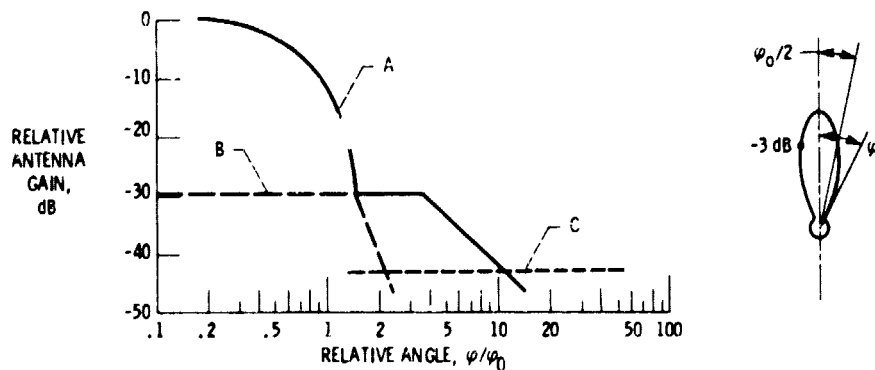


Figure 4. - Reference patterns for co-polar and cross-polar components for a single-feed satellite transmitting antenna producing a beam of circular or elliptical cross-section.

- CURVE A: Co-polar component (dB)  
 $-12 \log(\phi/\phi_0)^2$  for  $0 \leq \phi \leq 1.58 \phi_0$   
 $-30$  for  $1.58 \phi_0 < \phi \leq 3.16 \phi_0$   
 $-(17.5 + 25 \log(\phi/\phi_0))$  for  $3.16 \phi_0 < \phi$   
 AFTER INTERSECTION WITH CURVE C: AS CURVE C
- B: Cross-polar component (dB)  
 $-30$  for  $0 \leq \phi/\phi_0 \leq 1.56$   
 $-(40 + 40 \log(\phi/\phi_0) - 1)$  for  $1.56 < \phi/\phi_0$   
 AFTER INTERSECTION WITH CURVE C: AS CURVE C
- C: (Minus the on-axis gain (dB))

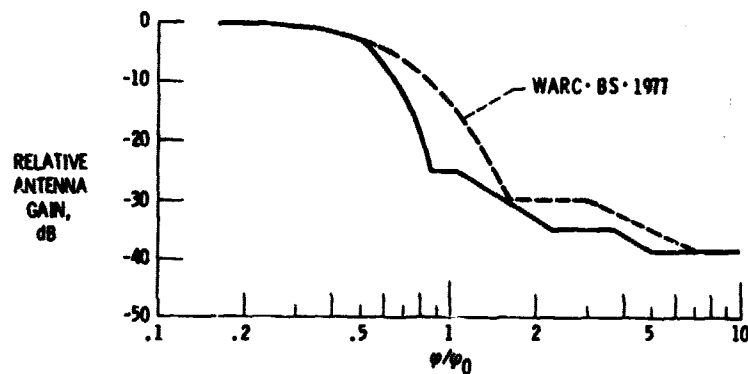


Figure 5. - Shaped-beam co-polar reference pattern.

$$G_0 = 38.8 \text{ dB}$$

$$\phi_0 = 2^\circ$$

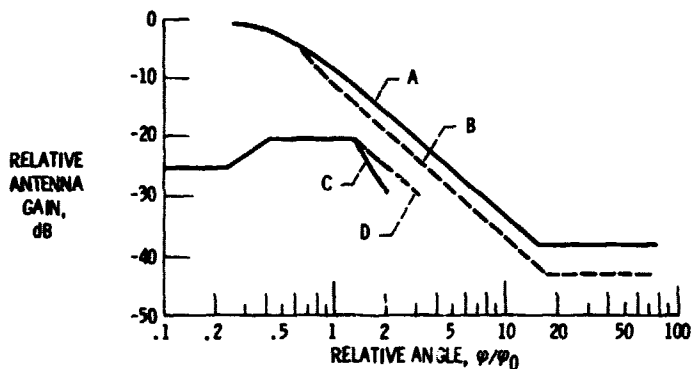


Figure 6. - Reference patterns for co-polar and cross-polar components for receiving antennas for individual reception in Region 2.

Antenna diameter:  $D = 1 \text{ m}$   
Half-power beamwidth:  $\phi_0 = 1.8^\circ$   
Nominal on-axis gain:  $G_0 = 40.2 \text{ dB}$

CURVE A: Co-polar component without side-lobe suppression (WARC-BS-77)

B: Co-polar component without side-lobe suppression (suggested)

$$\begin{aligned} 0 & \text{ for } \phi \leq 0.25\phi_0 \\ -12\phi/\phi_0^2 & \text{ for } 0.25\phi_0 < \phi \leq 0.94\phi_0 \\ -[11.3 + 25 \log(\phi/\phi_0)] & \text{ for } 0.94\phi_0 < \phi \leq 18.88\phi_0 \\ -43.2 \text{ dB } (-3 \text{ dB}) & \text{ for } \phi > 18.88\phi_0 \end{aligned}$$

C: Cross-polar component (WARC-BS-77)

D: Cross-polar component (suggested)

$$\begin{aligned} -25 & \text{ for } 0 \leq \phi < 0.25\phi_0 \\ -[30 + 40 \log(|\phi/\phi_0| - 1)] & \text{ for } 0.25\phi_0 < \phi \leq 0.44\phi_0 \\ -20 & \text{ for } 0.44\phi_0 < \phi \leq 1.28\phi_0 \\ -[17.3 + 25 \log(|\phi/\phi_0|)] & \text{ for } 1.28\phi_0 < \phi \leq 3.22\phi_0 \\ -30 & \text{ until intersection with co-polar component curve; then as} \\ & \text{ for co-polar component} \end{aligned}$$

Note 1. - The flat portion of the curves up to  $\phi/\phi_0 = 0.25$  takes account of the pointing error of the antenna.

Note 2. - These patterns should determine the levels exceeded by 10% of the side-lobe peaks beyond the first antenna side-lobe.

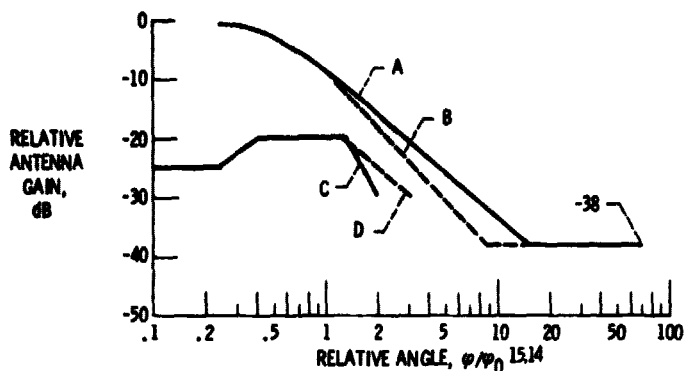


Figure 7. - Reference patterns for co-polar and cross-polar components for receiving antennas for individual reception in Region 2.

CURVE A: Co-polar component without side-lobe suppression (WARC-BC-77)

B: Co-polar component without side-lobe suppression (suggested)

$$\begin{aligned} &0 && \text{for } 0 < \varphi < 0.25 \varphi_0 \\ &-12\varphi + \varphi_0^2 && \text{for } 0.25 \varphi_0 < \varphi < 0.707 \varphi_0 \\ &-3.9 + 20 \log (\varphi / \varphi_0) && \text{for } 0.707 \varphi_0 < \varphi < 1.26 \varphi_0 \\ &-[8.5 + 30 \log (\varphi / \varphi_0)] && \text{for } 1.26 \varphi_0 < \varphi < 9.6 \varphi_0 \\ &-38 \text{ dB} && \text{for } \varphi > 9.6 \varphi_0 \end{aligned}$$

C: Cross-polar component (WARC-BS-77)

D: Cross-polar component (suggested) (same as Fig. 6)

Note 1. - The flat portion of the curves up to  $\varphi / \varphi_0 = 0.25$  takes account of the pointing error of the antenna.

Note 2. - These patterns should determine the levels exceeded by 10% of the side-lobe peaks beyond the first antenna side-lobe. However, derivation was done on the basis of peak values.

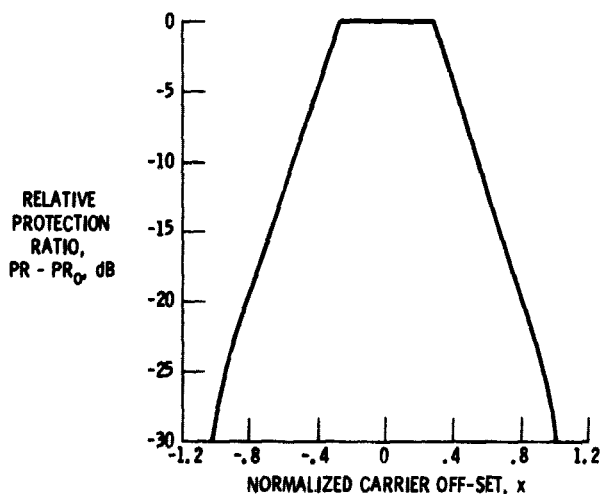


Figure 8. - Protection ratio relative to co-channel values for FMTV.

$$PR - PR_0 = \begin{cases} 0 & |x| \leq 0.274 \\ -35.6 (|x| - 0.274) & 0.274 < |x| \leq 0.92 \\ -23 - 71 (|x| - 0.92) & |x| > 0.92 \end{cases} \text{ dB}$$